

Calculation of Planetary Precession with Quantum-corrected Newton's Gravitation*

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Abstract

With consideration of quantization of space, we relate the Newton's gravitation with the Second Law of thermodynamics. This leads to a correction to its original form, which takes into consideration of the role of classical measurement. Our calculation shows this corrected form of gravitation can give explanation for planetary precession.

The most distinctive feature of quantum mechanics is the concept of measurement. It is more reasonable and closer to reality compared with that in classical physics. Therefore the first step that leads to a successful combination of quantum mechanics and general relativity should be the introduction of the role of measurement into classical physics. Such attempt has been scarcely seen because most physicists will not give up the concept of independent objectiveness of reality in classical physics. But as a matter of fact we can only talk about the part of the *reality* we are able to measure, which is certainly under the influence of our measurement. It is the purpose of this paper to show with an example that this philosophy of quantum physics may also work in classical physics.

We know in both classical and quantum physics, in reality and in philosophy, nothing can be made up of zeros, otherwise many paradoxes like Zeno paradox [1] would arise. Therefore it is natural and reasonable to assume that there must be a basic measuring unit in every single measurement, which can not be measured itself. It is the basic brick that constructs the result of our measurement. I would like to call it *uncertainty quantum*, since we are uncertain about its nature in principle. For example, in order for the concept of length to make sense, there must be a length quantum. And time would have no meaning if there were not a time quantum. Evidently this quantum is characteristic of an observer. In this way, we have introduced a subjective feature into classical physics. We

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shall see that with this simple correction, we can explain with some precision the planetary precession with Newton's' gravitation law, which formerly can not be calculated without Einstein's theory of relativity. Such attempt may not be meaningless when one consider all the futility in synthesizing Einstein's relativity and quantum theory.

For any distance L measured by an observer, there must be a space (or length) quantum q_l . L will be an integer if measured in q_l . Thus the system actually contains the following states: the two mass points separated by $L, L - q_l, L - 2q_l, \dots, q_l$. When we observe such system, we can only sense the overall gravitational effect of the system, rather than that of single mass point. That is, for a system of two mass points separated by a distance of L , we can not *sense gravitationally* any difference between these states. Therefore we can assume a state weight for the system: the weight for each state is $1/L$, where L is an integer. In the same way we know that the weight of every state in the system composed of two mass points separated by $(L - 1)q_l$ is $\frac{1}{L-1}$. From the famous Second Law of thermodynamics we know that the system should evolve to states with larger and larger statistical weights. That gives rise to gravitational interaction. As is usually done in nonequilibrium statistical physics[2], we presume that the intensity of this interaction should be proportional to the increment of the weight of the state. That is

$$F \propto \frac{1}{L-1} - \frac{1}{L} \quad (1)$$

This changes to the following form when we use common measuring unit

$$F = G(q_l)m_1m_2 \frac{1}{L(L - q_l)} \quad (2)$$

where $G(q_l)$ is gravitational constant and we are unable to give the exact theoretical expression for it now. When q_l is too small compared with the distance L , (2) changes to the familiar form of Newton's gravitation. But we shall see that it is this small approximation that wipes out the effect of measurement itself as well as, at least partly, the planetary precession.

The orbital equation for Newton's gravitation is [3]

$$\frac{d^2u}{d\theta^2} + u = \frac{k^2}{h^2} \quad (3)$$

where $r = \frac{1}{u}$ and θ are the polar coordinates of the planet. $k^2 = GM$, M is the mass of the sun, $h = \frac{2\pi ab}{\tau}$, a and b are long and short radius of the orbital ellipse, τ is period of the planet. When corrected gravitational formula (2) is employed, (3) changes to

$$\frac{d^2u}{d\theta^2} + (1 - \frac{q_l k^2}{h^2})u = \frac{k^2}{h^2} \quad (4)$$

planet	observation	relativity	$\delta = 0.01$	$\delta = 0.05$	$\delta = 0.0398$
Mercury	43.11 ± 0.45	43.03	10.8	54.1	43.08
Venus	8.4 ± 4.8	8.63	5.1	25.4	20.18
Earth	5.0 ± 1.2	3.84	3.1	15.4	12.30

Table 1: Calculation of planetary precession(in sec.)

in which terms of higher order of q_l are ignored. It is straightforward to get the solution of (4) :

$$r = \frac{p}{1 + Ap \cos x\theta} \quad (5)$$

where $p = \frac{h^2 - q_l k^2}{k^2}$, $x = \sqrt{1 - \frac{q_l k^2}{h^2}}$, A is an integral constant. The orbit described by (5) is also a periodical function with its period to be $\frac{2\pi}{x}$. The perihelion is at $\theta = \frac{2n\pi}{x}$, n is any integral. Therefore its centurial precession is

$$\triangle\theta = 2\pi\left(\frac{1}{x} - 1\right) \cdot \frac{100}{\tau} \quad (6)$$

The key point in this calculation is how to decide the space quantum q_l . We take the observation error [4] to be the uncertainty quantum. Judging from the decimal figure of the observed data, we see that the error for the precession is $\delta = 0.01'' \sim 0.05''$. This is not the systematic observation error, which may be at least partly responsible for the deviation of our calculation. So we get the space quantum in this way: $q_l = \delta b$, where b is the short radius of the planetary orbit. From the result of our calculation we can see the subjectively corrected Newton's gravitational law can give quite satisfactory result. Though it seems no better than general relativity, it takes into consideration of the function of measurement for the first time in Newton's dynamics.

There exists some deviation from the observation because there may be some minor and delicate difference between the exact meaning of observation error and that of the error in our space quantum correction. A mediate value $\delta = 0.398''$ gives good calculation for Mercury, but not so good for the others. This reasonably indicates that different space quanta underlie different measurements. Therefore we may anticipate that the amount of precession should decrease with the increasing precision of the *methods* of measurements in a certain range. This can be interpreted as due to the structural change in spacetime, or rather to the change of the uncertainty quantum involved in the observation. Such concept of measurement is consistent with quantum mechanics. The uncertainty principle in quantum physics actually tells us that the result of measurement depends on the ignorance of the variables that are not commutable with the one being measured. We hope that effort in unifying the concepts of measurement in classical and quantum physics may pave the way to the unification of the two realms.

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